Thermophoresis particle deposition – thermal radiation interaction on natural convection heat and mass transfer from vertical permeable surfaces

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Abstract

Purpose – The purpose of this paper is to study thermophoresis particle deposition and thermal radiation interaction on natural convection heat and mass transfer by steady boundary layer flow over an isothermal vertical flat plate embedded in a fluid saturated porous medium.

Design/methodology/approach – The governing partial differential equations are transformed into non-similar form by using special transformation and then the resulting partial differential equations are solved numerically by using an implicit finite difference method.

Findings – Different results are obtained and displaced graphically to explain the effect of various physical parameters on the wall thermophoresis deposition velocity and concentration profiles. It is found that the increasing of thermal radiation parameter or dimensionless temperature ratio heats the fluid and decreases temperature gradients near permeable wall, which increases local Nusselt numbers and decreases wall thermophoresis velocities. It is also found that the effect of power indices of either temperatures or concentration enhances both local Nusselt numbers and wall thermophoresis velocities. Comparison with previously published work in the limits shows excellent agreement.

Originality/value – The paper presents useful conclusions based on graphical results obtained from studying numerical solutions for thermophoresis-thermal radiation heat and mass transfer interaction by steady, laminar boundary layer over a vertical flat plate embedded in a porous medium.

Keywords Heat transfer, Mass transfer, Convection, Porous materials

Paper type Research paper

Nomenclature

C = species concentration
C_F = Forcheimer coefficient
C_f = local skin friction factor
c_p = specific heat capacity
D = Brownian diffusion coefficient
Da_x = local Darcy number, K/x^2
f = dimensionless stream function
g = gravitational acceleration
h = local heat transfer coefficient
K = permeability
k = thermal conductivity
k_t = thermophoresis coefficient
Le = Lewis number, α_m/D
p = constant defined in Equation (6)
$M_w(x)$ = local surface mass flux
N = buoyancy ratio, $[\beta_C(C_w - C_\infty)/\beta_T(T_w - T_\infty)]$

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N_t = dimensionless temperature ratio, $T_\infty/[T_w(x) - T_\infty]$

N_c = dimensionless concentration ratio, $C_\infty/[C_w(x) - C_\infty]$

Nu_x = local Nusselt number, hx/k

P = pressure

Pr = Prandtl number, υ/α

$q_w(x)$ = local surface heat flux

$Q_w(x)$ = dimensionless local surface heat flux

Ra_x = local Rayleigh number, $Kg\beta(T_w(x) - T_\infty)x/\upsilon\alpha$

R_d = thermal-radiation parameter, $4\sigma T_\infty^3/k(\alpha_r + \sigma_s)$

Sh_x = local Sherwood number, $M_w(x)x/D(C_w(x) - C_\infty)$

T = temperature

u, v = velocity components in x and y directions

V = porous wall suction or injection velocity

V_w = porous wall suction or injection velocity

v_t = thermophoresis velocity

v_{tw} = thermophoresis velocity at wall

V_t = dimensionless thermophoresis velocity, $v_t x/\alpha_m$

V_{tw} = dimensionless thermophoresis velocity at wall

x, y = axial and normal coordinates

Greek symbols

α_m = effective thermal diffusivity of the porous medium

α_r = Rosseland mean extinction coefficient

β_T = coefficient of thermal expansion, $(-1/\rho)(\partial\rho/\partial T)_P$

β_C = coefficient of concentration expansion, $(-1/\rho)(\partial\rho/\partial C)_P$

σ = Stefan–Boltzman coefficient

σ_s = scattering coefficient

ς = non-similarity parameter, $V\{4x/\upsilon^2 g\beta(T_w - T_\infty)\}^{1/4}$

η = pseudo-similarity variable

θ = dimensionless temperature

φ = dimensionless concentration

ε = porosity

μ = dynamic viscosity

υ = kinematic viscosity

ρ = fluid density

τ_w = local wall shear stress

ψ = dimensional stream function

Subscripts

w = surface conditions

∞ = free stream condition

t = thermophoresis effects

1. Introduction

Thermophoresis is a phenomenon, which causes small particles to be driven away from a hot surface and toward cold one, small particles, such as dust, when suspended in a gas temperature gradient, experience a force in the direction opposite to the temperature gradient. This phenomenon has many practical applications in removing small particles from gas streams, in determining exhaust gas particles trajectories from combustion devices, and in studding the particulate material deposition on turbine blades. It has been also shown that thermophoresis is the dominant mass transfer mechanism in the modified chemical vapor deposition process used in the fabrication of optical fiber performance. Also, it is important in view of its relevance to postulated accidents by radioactive particle deposition in nuclear reactors. In many industries, the composition of processing gases may contain any of an unlimited range of particle, liquid or gaseous contaminants and may be influenced by uncontrolled factors of temperature and humidity. In the application of pigments or chemical coating of metals, or removal of particles from a gas stream by filtration, there can be distinct advantages in expositing deposition mechanism to improve efficiency.

Goren (1977) studied the role of thermophoresis of an incompressible fluid over a flat plate in order to calculate deposition rates and it is found that the substantial changes in surface deposition can be obtained by increasing the temperature difference between the surface and free stream. Gokoglu and Rosner (1986) obtained a set of similarity solutions for the two-dimensional laminar boundary layers, Park and Rosner (1989) obtained another set of similarity solutions for the stagnation point flows. Chiou (1991) obtained the similarity solutions for a continuously moving surface in a stationary incompressible fluid, including the combined effects of convection, diffusion, wall velocity and thermophoresis. Garg and Jayaraj (1998) discussed the thermophoresis of small particles in forced convection laminar flow over inclined plates; Epstein *et al.* (1985) have studied the thermophoresis transport of small particles through a free convection boundary layer adjacent to a cold, vertical deposition surface in a viscous and incompressible fluid. Chiou (1998) has considered the particle deposition for natural convection boundary layers around an isothermal vertical cylinder. Convective flows in porous media have been extensively investigated during the last several decades, due to many practical applications. Comprehensive literature surveys concerning the subject of porous media can be found in the most recent books by Ingham and Pop (2002), Nield and Bejan (1999) and Pop and Ingham (2001).

However, the thermal radiation heat transfer effects on different flows are very important in high temperature processes and space technology. The effects of thermal radiation on heat transfer problems have been studied by Soundalgekar *et al.* (1960), Hossain and Takhar (1996) and Hossain *et al.* (1999a, b). Duwairi and Damseh (2004a, b) studied recently the effects of radiation heat transfer for the natural convection heat transfer problem form radiant vertical surfaces with fluid suction or injection, the non-similar parameter was found to be the fluid suction or injection through the porous surface. Also, Duwairi and Damseh (2004a, b) studied the thermal radiation-mixed convection interaction from vertical porous surfaces for the cases of both buoyancy aiding flow and buoyancy opposing flow with viscous dissipation included. It was found that thermal radiation increases heat transfer rates for both cases.

Due to the practical importance of thermophoresis, several authors have recently studied these phenomena. For example, Chamkha and Pop (2004) obtained a similarity solutions with approximation of thermophoresis parameter in derivation of governing equations. Duwairi and Damseh (2008) investigate the thermophoresis effects on mixed convection heat and mass transfer from non-isothermal vertical surfaces embedded in a porous media using Darcy law. Therefore, consideration in this work is given to the thermophoresis-thermal radiation interaction on natural convection heat and mass transfer problems from vertical surfaces with constant surfaces temperatures embedded in saturated porous medium, the full momentum equation is used in the transformation. The governing partial differential equations are transformed into non-similar form and then solved using box method as described by Cebecei and Bradshaw (1984). Numerical results for the velocity, temperature and concentration profiles as well as the thermophoresis velocity, local coefficient of friction and local Nusselt numbers under the effect of different dimensionless groups are presented.

2. Mathematical formulation

Consider natural convection heat and mass transfer from a permeable vertical surface embedded in saturated porous medium as shown in Figure 1. The analysis is carried out for the case of the constant wall temperature $T_w(\mathrm{x}) =$ constant and constant wall concentration $C_w(\mathrm{x}) =$ constant, the x coordinate is measured from the leading edge of the plate and the y coordinate is measured normal to the plate. The gravitational acceleration g is acting downward in the direction opposite to the x coordinate. The non-Darcy model is used in the analysis, also the properties of the fluid are assumed to be constant and the porous medium is treated as isotropic. Allowing for both Brownian motion of particles and thermophoresis transport, the governing equations can be written as Nield and Bejan (1999), Chiou (1998) and Duwairi (2000):

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{1}$$

$$\frac{u\partial u/\partial x + v\partial u/\partial y}{\varepsilon^2} + \frac{v}{K}u + \frac{C_F}{\sqrt{K}}u^2 = \frac{v}{\varepsilon}\frac{\partial^2 u}{\partial y^2} + g(\beta_T(T - T_\infty) + \beta_c(C - C_\infty)) \tag{2}$$

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \alpha_m\frac{\partial^2 T}{\partial y^2} + \frac{-16\sigma}{3\alpha_r\rho c_p}\frac{\partial}{\partial y}\left(T^3\frac{\partial T}{\partial y}\right) \tag{3}$$

$$u\frac{\partial C}{\partial x} + v\frac{\partial C}{\partial y} + \frac{\partial(Cv_t)}{\partial y} = D\frac{\partial^2 C}{\partial y^2} \tag{4}$$

The last term on the right side of the energy equation is the thermal radiation heat flux approximated using Rosseland diffusion equation, which is valid for thick boundary layers and so small scattering effects form other fluid layers to the selected one. The first and second terms on the left side of the mass concentration

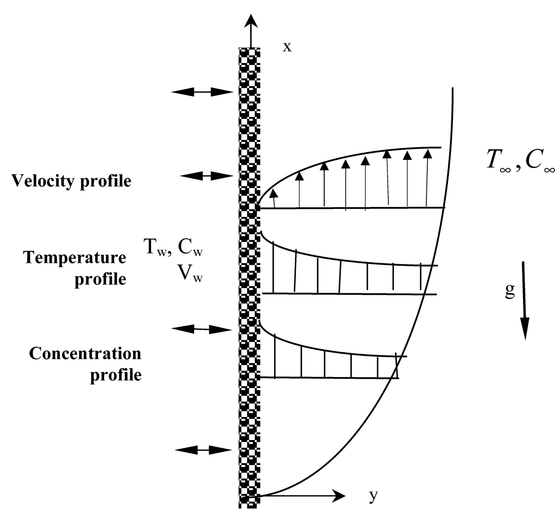

Figure 1. Natural convection heat and mass transfer over vertical permeable radiate flat plate embedded in fluid saturated porous medium

equation is the convective mass flux, while the third term is the thermophoresis mass flux. The u and v are the velocity components in x and y directions, respectively, T is the fluid temperature, C is the fluid concentration, K is the permeability of the porous medium, υ is the kinematic viscosity, D is the Brownian diffusion coefficient, α_m is the effective thermal diffusivity of the porous medium, α_r is the Roseland diffusion constant, ρ is the fluid density, c_p is the specific heat, σ is the Stefan–Boltzman constant, σ_s is the scattering coefficient, and β_T and β_C are the thermal expansion coefficient of temperature and concentration, respectively. The effect of thermophoresis is usually prescribed by means of the average velocity, which a particle will acquire when exposed to a temperature gradient. Under boundary layer approximations the temperature gradient in the y direction is very much larger than in the x direction, and therefore only the thermophoresis velocity in the y direction is considered. In consequence the thermophoresis velocity v_t can be expressed in the form:

$$v_t = -k_t \frac{\upsilon}{T}\frac{\partial T}{\partial y} \tag{5}$$

Here k_t is the coefficient of thermophoresis. The boundary conditions that describe the governing equations (1-5) can be written as:

$$\begin{aligned} &u = 0, T = T_w, C = C_w \quad \text{at } y = 0 \\ &u = 0, T = T_\infty, C = C_\infty \quad \text{at } y \to \infty \end{aligned} \tag{6}$$

The minus sign $V = -V_w$ correspond to the fluid suction or withdrawal and the plus sign $V = V_w$ corresponds to fluid blowing or injection. In this paper, both cases suction and injection will be treated. In order to transform the governing equations (1-6) from the (x, y) coordinates to the dimensionless coordinate (ς, η) the following non-dimensional variables are introduced:

$$\begin{aligned} &\eta = \frac{V}{\upsilon\varsigma}, \varsigma = \varsigma(x)\psi = V^{-3}\upsilon^2 g\beta(T_w - T_\infty)\varsigma^3\{f(\varsigma,\eta) + (\pm 1/4)\varsigma\} \\ &\theta(\varsigma,\eta) = \frac{T - T_\infty}{T_w - T_\infty},\ \varphi(\varsigma,\eta) = \frac{C - C_\infty}{C_w - C_\infty} \end{aligned} \tag{7}$$

The plus and minus signs correspond to suction and injection of the fluid, respectively. In the equations above, the stream function ψ satisfied the continuity equation (1) with $u = \partial\psi/\partial y$ and $v = -\partial\psi/\partial x$. Finally, one can obtain the following system of dimensionless equations:

$$\begin{aligned} &\frac{f'''}{\varepsilon} + \frac{3ff'' - 2f'^2 + (\pm\varsigma f'')}{\varepsilon^2} + K_1 f' + K_2 f'^2 + \theta + N\varphi \\ &\quad = \frac{\varsigma\left(f'\frac{\partial f'}{\partial\varsigma} - f''\frac{\partial f}{\partial\varsigma}\theta'\right)}{\varepsilon^2} \end{aligned} \tag{8}$$

$$\frac{1}{\Pr}\left\{\theta'' + \frac{4R_d}{3}\left([1 + (\theta_w - 1)\theta]^3\theta'\right)'\right\} + 3f\theta' + (\pm\varsigma\theta') = \varsigma\left(f'\frac{\partial\theta}{\partial\varsigma} - \theta'\frac{\partial f}{\partial\varsigma}\right) \tag{9}$$

$$\frac{1}{Le}\varphi'' + 3f\varphi' + (\pm\varsigma\varphi') + \frac{k_t \Pr}{\theta + N_t}\left[\theta'\varphi' + (\varphi + N_c)\theta'' - \frac{\varphi + N_c}{\theta + N_t}\theta'^2\right]$$
$$= \varsigma\left(f'\frac{\partial\varphi}{\partial\varsigma} - \varphi'\frac{\partial f}{\partial\varsigma}\right) \qquad (10)$$

With the corresponding boundary conditions:

$$f'(\varsigma,0) = 0, f(\varsigma,0) = 0$$

$$\theta(\varsigma,0) = 1, \varphi(\varsigma,0) = 1, f'(\varsigma,\infty) = 0, \theta(\varsigma,\infty) = 0, \varphi(\varsigma,0) = 0 \qquad (11)$$

Here $\varsigma(x) = V\{4x/\upsilon^2 g\beta(T_w - T_\infty)\}, \Pr = \upsilon/\alpha_m, Le = \alpha_m/D$, $N_c = C_\infty/[C_w(x) - C_\infty]$, $N_t = T_\infty/[T_w(x) - T_\infty], K_1 = \upsilon^2/KV^2, K_2 = C_F/Da_x$, $Ra_x = g\beta[T_w(x) - T_\infty]Kx/\upsilon\alpha, R_d = 4\sigma T_\infty^3/k(\alpha_r + \sigma_s), \theta_w = T_w/T_\infty = (1/N_t) + 1$, $Da_x = K/x^2 N = \beta_C(C_w(x) - C_\infty)/\beta_T(T_w(x) - T_\infty)$ and the primes denotes partial differentiations with respect to η. Note that for the case of $\varsigma = 0$ and $N_c = 0$, the governing equations (8-10) with the corresponding boundary conditions, Equation (11) are reduced to those obtained by Chamkha and Pop (2004), for the case of pure free convection, where a similarity solutions are obtained. Some of the physical quantities of practical interest include the velocity components u and v in the x and y directions, respectively, the local Nusselt number $Nu_x = hx/k$ and the dimensionless wall thermophoresis deposition velocity V_{tw}. They are given by:

$$\frac{u}{g\beta V^{-2}\upsilon(T_w - T_\infty)} = \varsigma^2 f'(\varsigma,\eta) \qquad (12)$$

$$\frac{v}{V} = -(1/\varsigma)\left(3f(\varsigma,\eta) - \eta f'(\varsigma,\eta) + (\pm\varsigma) + \varsigma\frac{\partial f(\varsigma,\eta)}{\partial\varsigma}\right) \qquad (13)$$

$$Nu_x = \frac{q_w(x)\upsilon}{kV(T_w - T_\infty)} = -(1 + (4/3)R_d\theta_w^3)\varsigma^{-1}\theta'(\varsigma,0) \qquad (14)$$

and

$$V_{tw} = -\frac{k_t \Pr}{1 + N_t}\theta'(\varsigma,0) \qquad (15)$$

Notice also that for the case of $k_t = 0$ (absence of thermophoresis), $\varepsilon = 1$ (plain medium) the equations are reduced to those of (Duwairi and Damseh, 2004a, b).

3. Numerical solution

The partial differential equations (8-10) under the boundary conditions (11) are solved numerically by using an implicit iterative tridiagonal finite-difference method Cebeci and Bradshaw, (1984) and Keller (1978). In this method, any quantity g at point (ς_n, η_j) is written as g_j^n. Quantities and derivatives at the midpoints of grid segments are approximated to second order as:

$$g_j^{n-1/2} = \frac{1}{2}\left(g_j^n + g_j^{n-1}\right), \quad g_{j-1/2}^n = \frac{1}{2}\left(g_j^n + g_{j-1}^n\right) \tag{16}$$

$$\begin{aligned}\left(\frac{\partial g}{\partial \varsigma}\right)_j^{n-1/2} &= \Delta\varsigma^{-1}\left(g_j^n - g_j^{n-1}\right), (g')_{j-1/2}^n \\ &= \Delta\eta^{-1}\left(g_j^n - g_{j-1}^n\right)\end{aligned} \tag{17}$$

Where g is any dependent variable and n and j are the node locations along the ς and η directions, respectively. First, the third-order partial differential equation is converted in a first order by substitutions $f' = s$, and $s' = w$, the difference equations that are to approximate the previous equations are obtained by averaging about the midpoint $(\varsigma_n, \eta_{j-1/2})$, and those to approximate the resulting equations by averaging about $(\varsigma_{n-1/2}, \eta_{j-1/2})$. At each line of constant ς , a system of algebraic equations is obtained. With the nonlinear terms evaluated at the previous station, the algebraic equations are solved iteratively. The same process is repeated for the next value of ς and the problem is solved line by line until the desired ς value is reached. A convergence criterion based on the relative difference between the current and previous iterations is employed. When this difference reaches 10^{-5} , the solution is assumed to have converged and the iterative process is terminated. The effect of the grid size $\Delta\eta$ and $\Delta\varsigma$ and the edge of the boundary layer η_∞ on the solution had been examined. The results presented here are independent of the grid size and the η_∞ up to the fourth decimal point.

This method is used successfully by Gorla and Kumari (1999) to solve for free convection of non-Newtonian fluid along vertical surfaces. Hossain *et al.* (1999a, b) used this method also to study convection heat transfer of micropolar fluid along inclined surfaces. Duwairi (2005) used this method to solve non-similar thermal radiation–convection interaction inside boundary layers and Chamkha and Al-Humoud (2007) used this method to study mixed convection heat and mass transfer of a non-Newtonian fluid. The accuracy of the selected method was tested by comparing the results with those of the classical natural-convection problem over a vertical isothermal impermeable plate by Oosthuizen and Naylor (1999). Table I shows a comparison between the Nusselt numbers at different mixed convection parameter, ς obtained by

Pr	Oosthuizen and Naylor (1999) $-\theta'(0,\varsigma)$	Current study $-\theta'(0,\varsigma)$
0.01	0.0180	0.018025
0.03	0.0400	0.040023
1	0.4010	0.401113
2	0.6025	0.602492
100	4.8984	4.898421

Table I. Values of $-\theta'(\zeta, 0)$ at selected values of *Pr* and absence of mass transfer for $\varsigma = 0$, $K_1 = 0$, $K_2 = 0$, $\varepsilon = 1$ compared to those obtained by Oosthuizen and Naylor (1999) the presented numerical method and that of the mentioned reference for the case of absence of mass transfer and thermophoresis deposition effects. It is seen that the present results are in a good agreement. This favorable comparison lends confidence in the numerical results to be reported in the next section.

4. Results and discussion

In this paper the thermophoresis-thermal radiation interaction is studied. The effect of thermophoresis is incorporated in the governing equations by the inclusion of k_t (thermophoresis coefficient). The effect of thermal radiation is incorporated in the governing equations by the inclusion of radiation parameter $R_d = 4\sigma T_\infty^3/k(\alpha_r + \sigma_s)$.

Figure 2 shows the velocity profiles for selected values of $Pr = 1.0$, $Le = 10$, $k_t = 0.2$, $K_2 = 0.1$, $K_1 = 2$, $N = 5$, $N_t = 50$, $N_c = 10$ and different porosity, radiation parameter, and suction or injection parameter. As the porosity is increased the velocity inside the boundary layer are increased; this is due to small effect of the solid matrix. As the thermal-radiation parameter is increased the velocities are also increased; this is

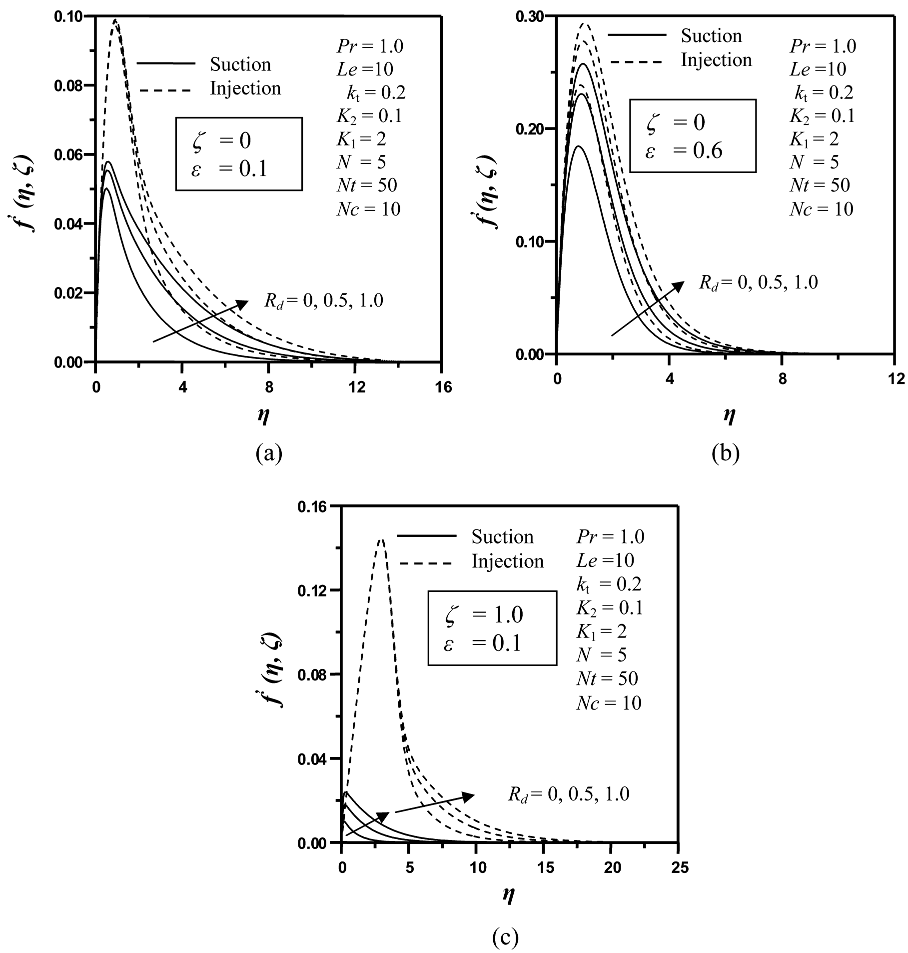

Figure 2.
Dimensionless velocity profiles for different values of porosity, thermal radiation parameter and fluid suction or injection parameters

due to favorable buoyancy forces. It is also clear that as the suction parameter is increased the velocities inside the hydrodynamic boundary layer are decreased and as the injection parameter is increased the velocities are increased in the presence of fluid injection.

Figure 3 shows the temperature profiles for selected values of $Pr = 1.0$, $Le = 10$, $k_t = 0.2$, $K_2 = 0.1$, $K_1 = 2$, $N = 5$, $N_t = 50$, $N_c = 10$ and different porosity, radiation parameter and suction or injection parameter. It is clear that as the radiation parameter is increased the thermal boundary layers are broadened and the temperatures inside the boundary layer are increased; this is due to excessive heating of fluid layers by thermal radiation heat fluxes. As the porosity is increased the thermal boundary layer thicknesses are decreased, which leads to higher temperature gradients. It is also clear that as the suction parameter is increased the thermal boundary layer thicknesses are decreased, and as the injection parameter is increased the thermal boundary layer thicknesses are increased.

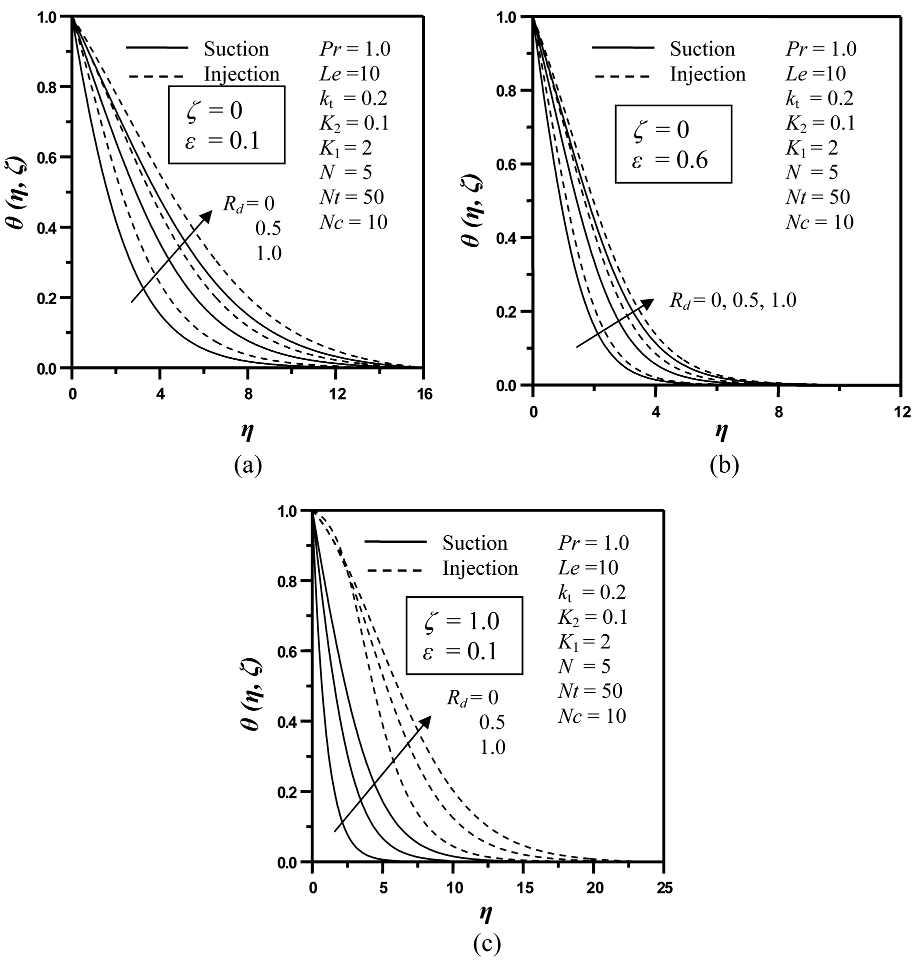

Figure 3.
Dimensionless temperature profiles for different values of porosity, thermal radiation parameter and fluid suction or injection parameters

Figure 4 shows the concentration profiles for selected values of $Pr = 1.0$, $Le = 10$, $k_t = 0.2$, $K_2 = 0.1$, $K_1 = 2$, $N = 5$, $N_t = 50$, $N_c = 10$ and different porosity, radiation parameter and suction or injection parameter. It is clear that as the radiation parameter is increased the concentration boundary layers are thinned and the mass fluxes are increased; this is due to favorable thermophoresis mass flux near radiate permeable vertical wall. As the porosity is increased the concentration boundary layer thicknesses are decreased, which leads to higher concentration gradients, and consequently mass fluxes. It is also clear that as the suction parameter is increased the thermal boundary layer thicknesses are decreased, and as the injection parameter is increased the thermal boundary layer thicknesses are increased.

Figures 5-7 show the velocity, temperature and concentration profiles for selected values of $Pr = 1.0$, $Le = 10$, $k_t = 0.2$, $K_2 = 0.1$, $K_1 = 2$, $N = 5$, $N_t = 50$, $N_c = 10$ and different temperature parameters, and suction or injection parameters. The increasing of temperature parameter decreased velocities inside the hydrodynamic boundary

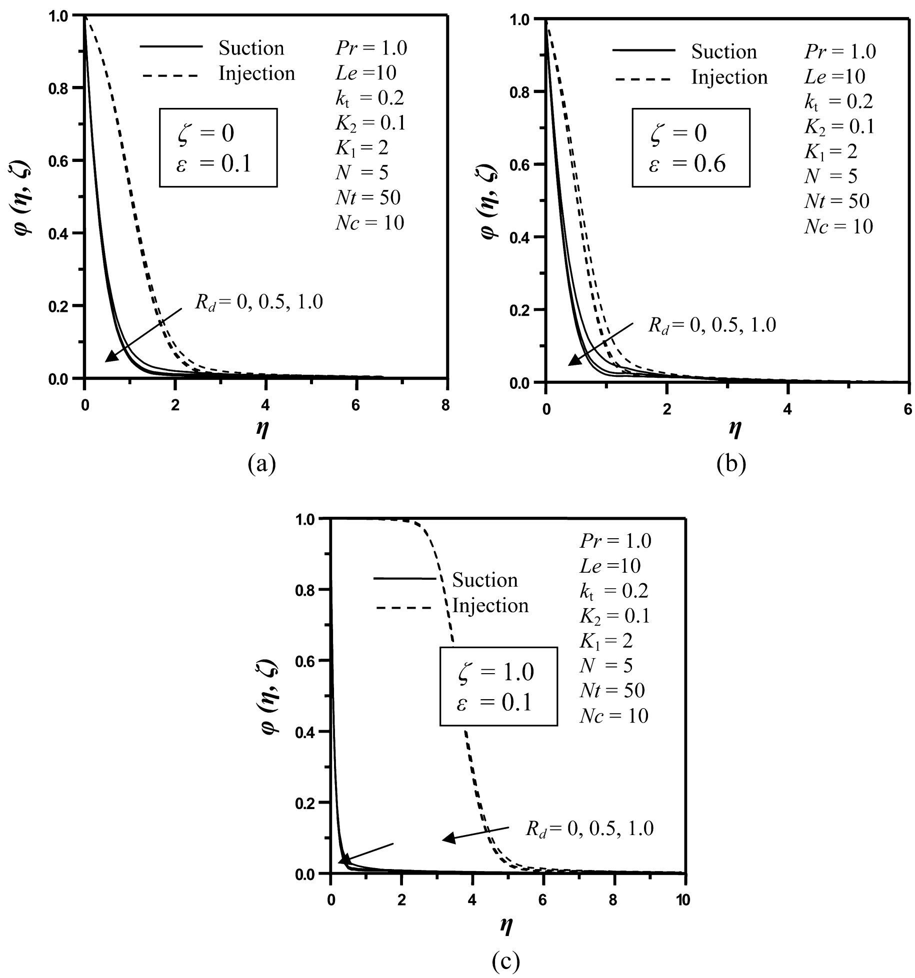

Figure 4.
Dimensionless concentration profiles for different values of porosity, thermal radiation parameter and fluid suction or injection parameters

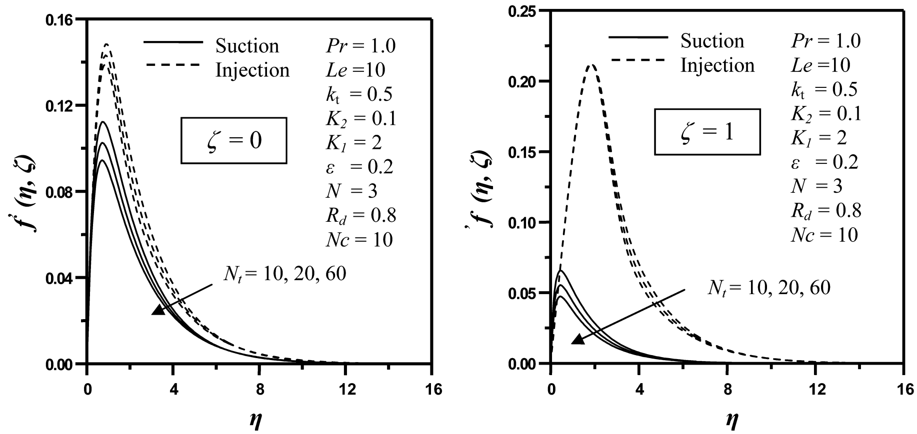

Figure 5.
Dimensionless velocity profiles for different values of temperature parameter and fluid suction or injection parameters

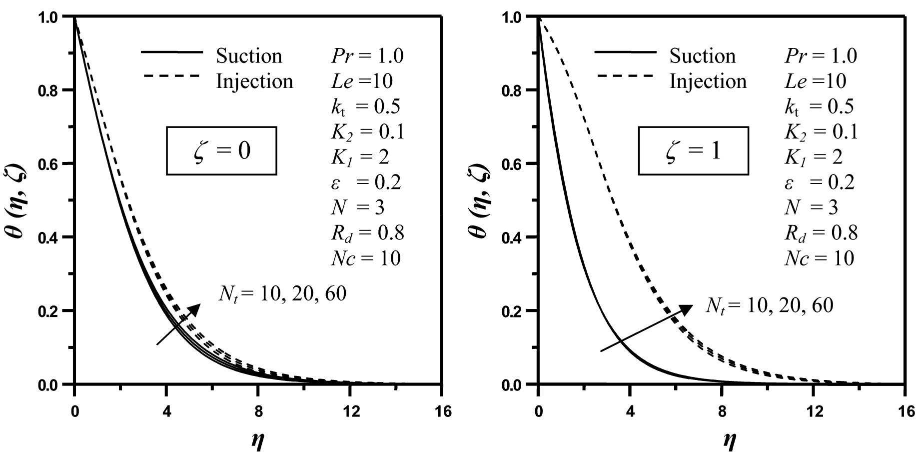

Figure 6.
Dimensionless temperature profiles for different values of temperature parameter and fluid suction or injection parameters

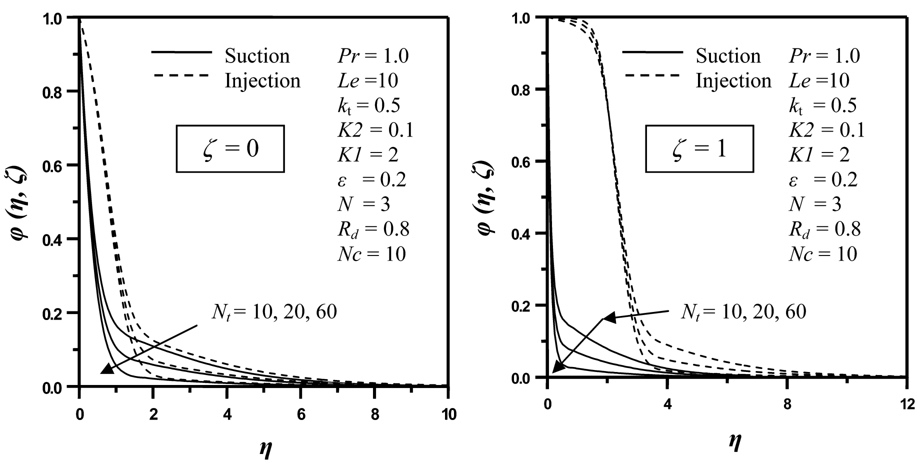

Figure 7.
Dimensionless concentration profiles for different values of temperature parameter and fluid suction or injection parameters

layer; this is due to small favorable buoyancy forces, and consequently increased temperatures inside thermal boundary layers and broadened boundary layer thicknesses. Finally, the increasing of temperature parameter decreased concentration inside boundary layers and consequently increased mass fluxes.

Figure 8 shows the thermophoresis velocities for selected values of $Pr = 1.0, Le = 10$, $k_t = 0.2$, $K_2 = 0.1$, $K_1 = 2$, $N = 5$, $N_t = 50$, $N_c = 10$ and different temperature parameters, radiation parameter and suction or injection parameters. The increasing of temperature parameter decreased thermophoresis velocities inside the hydrodynamic boundary layer; this is due to small favorable buoyancy forces, the increasing of radiation

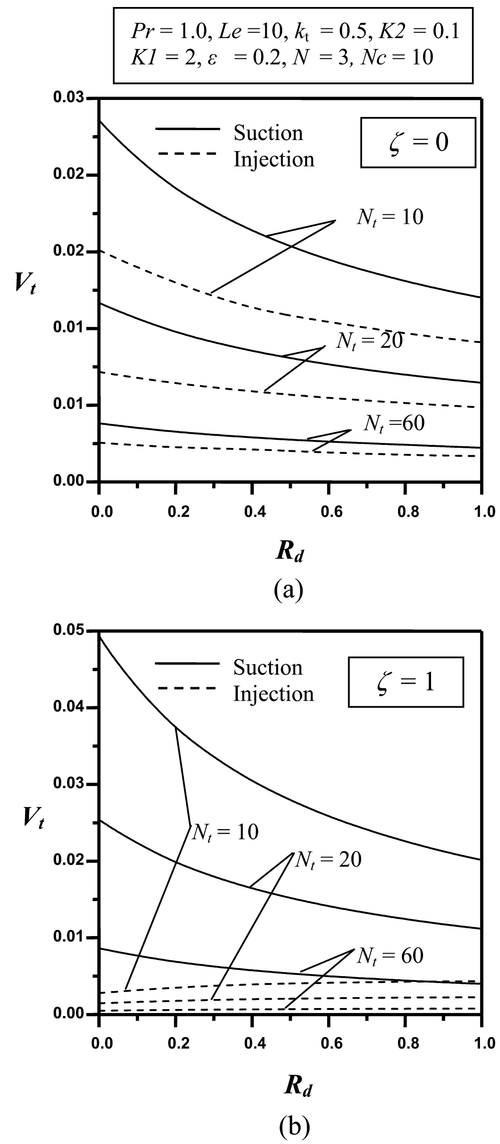

Figure 8.
Dimensionless thermophoresis velocities for different values of temperature parameter, radiation parameter and fluid suction or injection parameters

parameter decreased thermophoresis velocities also; this is due to excessive heating of fluid layers and small temperature gradients and consequently small values of thermophoresis velocities.

Figure 9 shows the local Nusselt numbers for selected values of $Pr = 1.0$, $Le = 10$, $k_t = 0.2$, $K_2 = 0.1$, $K_1 = 2$, $N = 5$, $N_t = 50$, $N_c = 10$ and different temperature parameters, radiation parameter and suction or injection parameters. The increasing of temperature parameter decreased local Nusselt numbers; this is due to small favorable buoyancy forces, and the increasing of radiation parameter increased local Nusselt numbers; this is due to favorable radiation heat fluxes between fluid layers.

5. Conclusions

Numerical solutions for thermophoresis-thermal radiation heat and mass transfer interaction by steady, laminar boundary layer over a vertical flat plate embedded in a

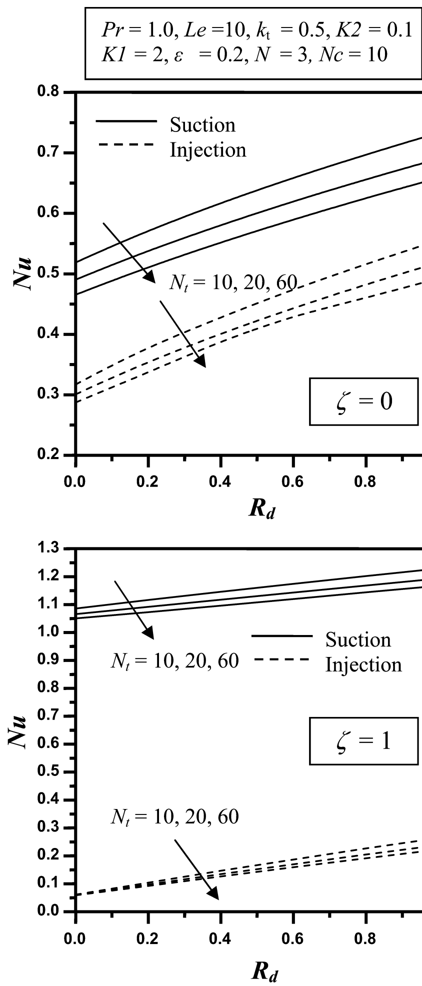

Figure 9.
Dimensionless Nusselt numbers for different values of temperature parameter, radiation parameter and fluid suction or injection parameters

porous medium were studied. Based on the obtained graphical results, the following conclusions were deduced:

- For both fluid suction or injection cases, the increased of thermal radiation parameter increased both velocity and temperatures and decreased species concentration inside the velocity, temperature and concentration boundary layers, respectively; this is due to excessive heating of fluid layers and higher values of buoyancy forces, which results in smaller values of mass fluxes.
- For both cases of fluid suction or injection, the increasing of dimensionless temperature ratio decreased both velocity and concentration inside boundary layers and broadened temperatures; this is due to small temperature differences and consequently smaller buoyancy forces inside boundary layers.
- The local Nusselt number values are enhanced when temperature difference between surface and free stream conditions or thermal radiation parameter are increased; this is due to favorable heat transfer mechanism of radiation between fluid layers and between fluid layer and permeable wall.
- The thermophoresis wall velocities are decreased when thermal radiation parameter or dimensionless temperature ratios are increased; this is due to excessive heating of fluid layers and consequently small temperature gradients near wall which leads to small thermophoresis mass immigration of fluid toward boundary layers.

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